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Times Varying Spectral Coherence Examination of Consumer Price Indices in Pakistan: A Wavelet Transform

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Abstract

Aim of this study is to examine the coherence of consumer price indices (CPI) variants in Pakistan using time series data. The techniques of data analysis are descriptive statistics and wavelet analysis. Plots of CPI variants show more frequent changes as compared to the base year / month from January 1990 to January 2008 and comparatively minor fluctuation subsequently. Wavelet power spectra of CPI General Index (CPI-Gen), CPI Food MoM (CPI-FMoM), CPI General YoY (CPI-GenYoY), and CPI Food YoY (CPI-FYoY) show weak correlation between wavelets and mother wavelet at low frequency bands, and vice versa at high frequency bands in sample period. In CPI Food Index and CPI General MoM, there is very strong correlation between the mother and daughter wavelets. Cross wavelet spectra show that CPI-General vs CPI-Food, CPI General vs CPI General (YoY), and CPI Food vs CPI Food (YoY)) at low frequency bands have weak co-movements, whereas, that is strong at high frequency bands. Cross wavelet spectra of CPI-General vs. CPI General (MoM) and CPI-Food vs. CPI-Food (MoM, at high bands have very strong co-movement. Wavelet coherence spectra show that at low frequency bands there is high coherence and correlation among variables, whereas, that is relatively low at high frequency bands. Wavelet coherence spectra as contained of CPI-General vs. CPI General (MoM), and CPI-Food vs. CPI Food (MoM) at high frequency bands show very weak coherence and correlation and the results also show that both the variables are in phase at most of the frequency and time resolutions.

Keywords: Consumer price index, CPI, time series, Wavelet, Pakistan.

1. Introduction

Consumer Price Index (CPI) is one of the very widely used measure of inflation and cost of living. It is used governments, businesses, investors, customers etc. for making economic decisions. CPI is a measure of average change in prices paid by a consumer over the period of time for a regular bucket of goods and services (Barkan et al., 2023). It tries

to measure the average cost of living by approximating the purchasing power of a single unit of currency in a given country. Precise predictions of CPI fluctuations are imperative for a country's economic development (Nguyen et al., 2023) as it is one of the most significant macroeconomic indicators for determining inflation (Cogoljević et al., 2018). In this context, Wang and his colleague made an attempt to calculate the CPI, they further claimed that their model provides a more accurate forecast of future CPI (Wang et al., 2012). Similarly, for getting more insights, Riofrío et al. (2020) carried a comprehensive comparative study of state-of-the-art predictive models from 2005 to 2019 including layers machine learning using support vector regression, neural networks approach using a sequential model (with long-short term memory) as well as classical approaches such as exponential smoothing and SARIMA. Bilgili et al. (2020) applied wavelet analysis to determine the impact of biofuel production on food prices in Unites States for the monthly time period during 1981-2018. They found a significant relationship between them in both short term and long-term cycles. Tiwari et al. (2013) used wavelet analysis method to examine the relationship between CPI and Producer Price Index (PPI) for Romania by using monthly data during the period of 1991 to 2011 in order to gauge the causality between them. They reported strong evidence of cyclical effects from variables, while there is no anti-cyclical effects observed.

The time series analysis of economic variables helps organizations to understand the underlying causes of trends and patterns over-time. There are lesser studies on analysis of CPI indices in Pakistan. There are many limitations in existing literature e.g. focusing static time-frequency analysis, failure to appreciate non-linear dynamics in price transmission, aggregate-level bias masking regional disparities, short-term fluctuations overshadowing structural shifts, and unrealistic assumption of homoscedasticity. To be more precise, in contemporary research literature (in general) and about Pakistan's CPI (particularly) there are several methodological weaknesses (Hanif et al., 2023; Hussain & Saeed, 2023; Yousaf & Ali, 2023). The authors could not find any study on CPI that examined the time and frequency domain simultaneously.

There is a dire need of examining the CPI indices in time and frequency domain. Therefore, research problem under investigation in this study is analysis of different variants of indices of CPI. The research objectives of the research are: i) to plot and inspect the CPI General Index (Spliced Base 2007-08=100), CPI Food Index (Spliced Base 2007-08=100), CPI General MoM, CPI Food MoM, CPI General YoY, and CPI Food YoY, ii) to evaluate the CPI General Index (Spliced Base 2007-08=100), CPI Food Index (Spliced Base 2007-08=100), CPI General MoM, CPI Food MoM, CPI General YoY, and CPI Food YoY on wavelet power spectrum, iii) to investigate the cross correlations of time series of different variants of CPI indices and iv) to evaluate coherence of the series of different variants of CPI indices over time. The common methods used for this type of analysis include correlation coefficient, GARCH models, Copula models, wavelet model, squared coherence wavelet approaches, Fama-French three-factor model, etc. The authors find wavelet transform as the most appropriate alternative model to obtain the objectives of the

study because the wavelet coherence analysis can address weaknesses of contemporary literature by leveraging its time-frequency decomposition capabilities.

Wavelet analysis is widely used methodology in different fields viz: neuroscience, wind engineering, industrial aerodynamics, finance, biology, medicine, economics (Ramsey, 1999), meteorology, energy, computer science, engineering, electronics & telecommunication, management sciences, social sciences, acoustics, geodynamics, astro/laser physics (Bloomfield et al., 2004; Donner & Thiel, 2007; Kelly et al., 2003; Li et al., 2009) and nuclear science, signal processing, engineering, medical sciences (Hassan et al., 2010; Qassim et al., 2013; Scholkmann & Dommer, 2012), astronomy, project management, energy, manufacturing, engineering, research & development, insurance, oil & gas, transportation, geology, geo-physics (Grinsted et al., 2004; Liu, Hsu & Grafarend, 2005; Zamani et al., 2013), oceanology, music, and environmental science etc. Within the wavelet transform, there is a wide variety of wavelets. There are two broad categories of wavelets i.e. discrete wavelets and continuous wavelets. Discrete wavelets include Beylkin, Coiflets, Daubechies wavelet, Haar, Vaidyanathan filter, Symmlets etc. Continuous wavelets include Mexican hat, Hermitian hat, Morlet, Addison, Hilbert-Hermitian wavelets etc.

The study uses Morlet wavelet transform one of the forms of continuous wavelet transform along with descriptive statistics of the variables. Wavelet analysis has advantage over other methods because it extracts local spectral and temporal information of simultaneously particularly when the classical assumptions of time series do not hold. It is very appropriate technique for analysis of non-stationary, non-linear, non-independent, and fractal trends of time series (Aloui & Hkiri, 2014; Aloui, Hkiri & Nguyen, 2016; Aloui, Hkiri & Yarovaya, 2018; Firouzi & Wang, 2021; Madaleno & Pinho, 2012; Reboredo et al., 2017; Rua & Nunes, 2009; Yang et al., 2017). The remainder of this article is organized as: literature review that briefly reviews the related literature on the cryptocurrency market; methodology that presents the methodology used in the study; data analysis, results, discussion and conclusion that discuss the research's results and conclusions, respectively.

2. Literature Review

There is burgeoning research on the variables under study worldwide. It is appropriate to explore the current literature to define the topic and provide an appropriate context for the study, therefore, the authors explored the research data bases of Wiley Online Library, Frontiers, Taylor & Francis Online, Ebsco Host, Springer Link, DOAJ, Emerald Insight, Elsevier (Science Direct), MDPI, and JStor. Using google as search engine and the keywords viz consumer price index, CPI, CPI in Pakistan, wholesale price index, price indices of Pakistan, CPI of Pakistan, wavelet, etc. the authors obtained many studies.

The studies have been reviewed critically from view point of importance and relevance. Highly relevant studies are reported here for setting the context of the study. Aguiar-Conraria and Soares (2011) employed wavelet analysis to study the time-frequency effect

of changing in oil prices on macro economy. Aguiar-Conraria et al. (2008) deployed wavelet analysis and highlighted that monetary policy and macro-economic variables have evolved and changed over the time. Miśkiewicz (2010) applied entropy correlation distance method to inspect the effect of Euro introduction on CPI. De and Mallik (2024) conducted research to scale the impact of global crude oil price on manufacturing output in India and revealed that there is a substantial positive relationship between CPI and Manufacturing Index (MI) and an inverse relationship between crude oil price with MI. Ngoc and Kriengsinyos (2022) investigated the relative time trends of prices of non-core i.e. less-healthy and core i.e. healthy using Thai CPI in Thailand from 2002 to 2021 to mitigate the development of obesogenic food environments in the country.

Parker (2024) provides a global database for energy CPI that covers a widespread range of advanced and developing economies. Ozpolat (2020) carried a comprehensive study to evaluate the importance of CPI and Producer Price Index (PPI) and concluded that both have bilateral long run causality in Central and Eastern European Countries (CEECs). Tiwari (2012a) delineated the unidirectional causality from consumer's price to producer's price in Australia and bolstered that consumer's price has a substantial predictive content in how the producer's price evolves. Similarly, Tiwari et al. (2014) analyzed the relationship between two inflation indices for Mexico and noticed bidirectional relationship among them. They further described that for longer time period (i.e. 8 to 32 months) PPI is leading CPI and for shorter time period (i.e. 1 to 7 months) CPI is the leading variable.

It is pertinent to document the efforts made by different researchers across the globe in the context of CPI and PPI viz: assessed the causal relationship between CPI and PPI in selected European countries (Akcay, 2011); examined between CPI and PPI that which drives which (Gang et al., 2009); identified the relationship between CPI and PPI in the Czech Republic (Khan et al., 2018); explored the determinants of PPI versus CPI inflation in emerging Asia (Jongwanich et al., 2019); investigated the effect of Chinese macro-economic declaration related to CPI and PPI on China security index (Liu et al., 2021); evaluated the relationship between CPI and PPI in Colombia (Martinez et al., 2013); exposed the irregularity in price transmission mechanism in Slovak (Rajcaniova & Pokrivcak, 2013); ascertained predictive content of PPI over CPI inflation in Mexico (Sidaoui et al., 2009); explored the relationship between CPI and PPI in Türkiye (Ulke & Ergun, 2014). Tiwari and Shahbaz (2013) endeavored to research the dynamic and static causality between consumers' price measured by CPI and producers' price measured by Whole-sale Price Index (WPI) in India and found bidirectional causality between CPI and WPI in both short and long run cases.

It is not out of place to mention some of the researches made in the domain of CPI and WPI across different sectors viz: affirmed the empirical evidence of causality from consumer prices to wholesale prices (Colclough & Lange, 1982); re-examined the evidence of wholesale and consumer prices inflation (Cushing & McGarvey, 1990); investigated the

relationship between consumer and wholesale prices (Guthrie, 1981); studied the causality between consumer and wholesale prices (Jones, 1986); assessed the empirical psychology between CPI and WPI in Pakistan (Shahbaz et al., 2010); observed the co-integration and causality between WPI and CPI in India (Tiwari & Shahbaz, 2010; Tiwari, 2012b).

3. Methodology

The study follows positivist research philosophy and deductive research approach. The design of the study, overall, is contemporary literature examination, data assortment, and analysis. Time series data of approximately thirty years (February 28, 1990 to November 30, 2019) of the variables namely CPI General Index (Spliced Base 2007-08=100), CPI Food Index (Spliced Base 2007-08=100), CPI General MoM, CPI Food MoM, CPI General YoY, and CPI Food YoY is obtained from central bank of Pakistan. The rationale of the sample period is the availability of the reliable time series data of selected variables. The selected variables capture critical multi-scale inflation dynamics e.g. isolating genuine price movements from structural breaks and disentangle short-term volatility from persistent inflationary trend. General vs. food CPI separation addresses sector-specific diversity reflecting Pakistan's inflation heterogeneity.

The methodology used is wavelet analysis (i.e. time plots and continuous wavelet transform of individual variables, cross wavelet transforms of all variant variables with CPI General Index (Spliced Base 2007-08=100), and wavelet coherence transform of all variant variables with CPI General Index (Spliced Base 2007-08=100). The study follows the classical procedure of Morlet wavelet transform (one of the forms of continuous wavelet transform) (Crowley & Mayes 2009; Rua & Nunes 2009). The wavelet transform is used to analyze time series and identify patterns changing over periodicity of the series. Wavelet analysis has advantage over other methods because it extracts local spectral and temporal information of simultaneously particularly when the classical assumptions of time series do not hold. It is very appropriate technique for analysis of non-stationary, non-linear, non-independent, and fractal trends of time series (Aloui et al., 2018; Firouzi & Wang, 2021; Madaleno & Pinho, 2012; Reboredo et al., 2017; Rua & Nunes, 2009; Yang & Fu, 2017). It addresses weaknesses of contemporary literature by leveraging its time-frequency decomposition capabilities. These type of the advantages of wavelet analysis over the contemporary techniques of econometrics provide strong rationale to opt is as methodological choice.

The analysis is performed on MatLab (the authors found it the most appropriate among considered options of R, Python, IDL-Fortran Code, FAWAV, LASTWAVE-C language routines, AWA3 etc.). Wavelet analysis is mathematical zoom-in zoom-out process. A wavelet means a wave-like oscillation which is localized in time and frequency both. It has two properties i.e. scale and location. Scale or dilation defines how "stretched" or "squished" a wavelet is that which is related to frequency. Location defines where the wavelet is positioned in time or space that which is related to time. A wavelet is a mathematical function used to

divide a given function or continuous-time signal into different scale components. It can assign a frequency range to each scale component and each scale component is then studied with a resolution that matches the scale. It is applied in wave propagation, data compression, signal processing, image processing, pattern recognition, computer graphics, and detection of aircraft, submarines, improvement of CAT scans, and other medical image techniques. The authors used it for pattern recognition. The schema of analysis, formulae, symbols, definitions, mathematical algorithm used in the study is adopted from Aziz et al. (2023).

4. Data Analysis and Results

This section of the study consists of data collection, descriptive & inferential analysis, explanation results of descriptive analysis & inferential analysis and discussion on the results qua realities.

4.1 Data

The data are obtained from the central bank of Pakistan. The variables include CPI General Index (Spliced Base 2007-08=100), CPI Food Index (Spliced Base 2007-08=100), CPI General MoM, CPI Food MoM, CPI General YoY, and CPI Food YoY. The time series is February 28, 1990 to 30th November, 2019 (Table 1).

Table 1: Time Series Dataset of CPI General Index and Some of Its Variants

Sr.	Series Name	Unit	30-Nov-19	31-Oct-19	-	-	31-Mar-90	28-Feb-90
1	CPI General Index (Spliced Base 2007-08=100)	2007-08 base year	263.59	260.46	-	-	23.85	23.70
2	CPI Food Index (Spliced Base 2007-08=100)	2007-08 base year	285.87	279.69	-	-	21.87	21.69
3	CPI General MoM	2007-08 base year	1.20	1.00	-	-	0.64	0.59
4	CPI Food MoM	2007-08 base year	2.21	1.22	-	-	0.83	0.68
5	CPI General YoY	2007-08 base year	12.29	11.08	-	-	6.06	5.68
6	CPI Food YoY	2007-08 base year	16.64	13.57	-	-	4.78	4.12

CPI is a common measure of inflation in a country that measures the change in the cost of a fixed basket of goods and services. It is used by governments to adjust monetary policy to meet inflation targets. The steady level of inflation is considered healthy for economic-growth of a country. This index is presented to the stakeholders in different variants as mentioned above. It is explained with its reference point (base year 2007-08 in this case), that of reference point always being 100 and the other years are compared with the base year and increase is measured as compared to. There are different sources of data on trading economics of Pakistan viz Pakistan Bureau of Statistics (PBS), Bureau of Labor Statistics, and State Bank of Pakistan. The representations of the index are made in both month on month (MoM) and year on year (YoY) basis i.e. change is measured every month compared to base month and every year to base year.

4.2 Data Analysis

Analysis of the data has two parts i.e. descriptive (statistical) analysis (Table 2) and inferential (mathematical/wavelet) analysis Figure 1-4.

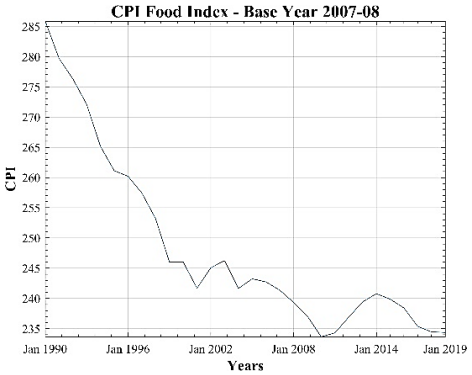
Table 2: Descriptive Statistics

	CPI General Index (Spliced Base 200708=100)	CPI Food Index (Spliced Base 200708=100)	CPI General MoM	CPI Food MoM	CPI General YoY	CPI Food YoY
Mean	104.9712694	110.0099624	0.678750674	0.737283281	8.36639027	8.912859196
Standard Error	3.621835721	4.12177333	0.04046823	0.073100471	0.231570908	0.322976112
Median	74.0982843	71.40486748	0.589664376	0.73927142	8.390348268	8.542421056
Mode	237.61	246.03	0.64	1.44	8.21	5.01
Standard Deviation	68.52834778	77.98761124	0.76569485	1.383125811	4.381527202	6.110994811
Sample Variance	4696.134449	6082.067507	0.586288604	1.91303701	19.19778062	37.34425757
Kurtosis	-0.978392215	-1.139360264	1.342977401	0.357451061	2.164340709	2.27164622
Skewness	0.67484926	0.653863464	0.662815405	0.112323996	1.041393068	1.138647785
Range	239.8910251	264.177162	5.70674411	8.26352468	24.79789413	34.42112489
Minimum	23.69897486	21.69283801	-1.323152555	-3.334063719	1.323971916	-0.579624302
Maximum	263.59	285.87	4.383591555	4.929460961	26.12186605	33.84150059
Sum	37579.71443	39383.56653	242.9927413	263.9474145	2995.167717	3190.803592
Count	358	358	358	358	358	358

Table 2 depicts the descriptive analysis of variables (i.e. it shows Mean, Standard Error, Median, Mode, Standard Deviation, Sample Variance, Kurtosis, Skewness, Range, Minimum, Maximum, Sum, and Count) of every variant of CPI under study. Wavelet (Figure 1-4) below is a mathematical function that provides modeling base for non-stationery time-series (signal). Wavelets provides information about time, location, and frequency by stretching, squeezing, and shifting. It is used for identifying and analyzing the discontinuities, patterns, sharp spikes, de-noising, and statistical estimation/prediction from economic time-series. Stepwise mathematical procedure is applied to the data i.e. choosing mother wavelet, multi-resolution analysis, decomposition, down-sampling, up-sampling, filtering, and summation. The frequency is broken into frequency-bands i.e. high-pass filter bands (coarser or approximate coefficients) and low-pass filters (detailed or precise coefficients). Wavelet by nature is visual representation of the analysis of time series. This mathematical process of integration and differentiation is performed on MatLab software that resulted into following set of plots and wavelet periodograms (scalograms). The times series' plots of all variables mentioned in Table 1, individual wavelet power spectra (continuous wavelet transform) of all variables, cross wavelet transform spectra of variables at Sr. 2-6 with variable at Sr. 1 in Table 1, and wavelet coherence spectra of variables at Sr. 2-6 with variable at Sr. 1 in Table 1 are generated and represented below as Figure 1-4 respectively. All plots and spectra have been inspected carefully by the authors for detection of horizontal or vertical patterns, sharp peaks, defuse areas, and other significant or non-significant events (if any) evident from the plots and spectra.

The findings of the inspection are given after each set of periodogram (i.e. under Figure 1-4 respectively).

Figure 1(a)



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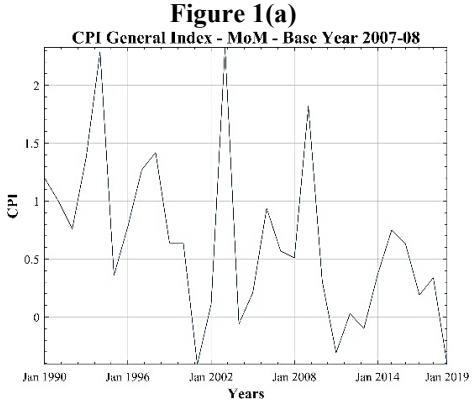


Figure 1(c)

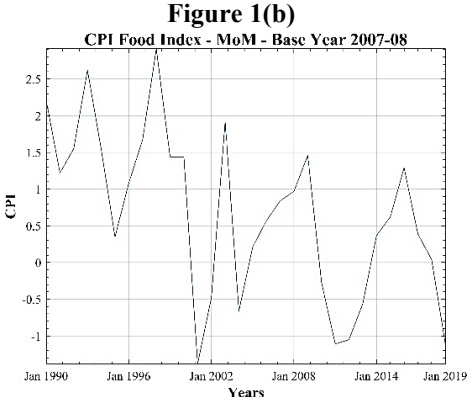


Figure 1(d)

Figure 1(e)

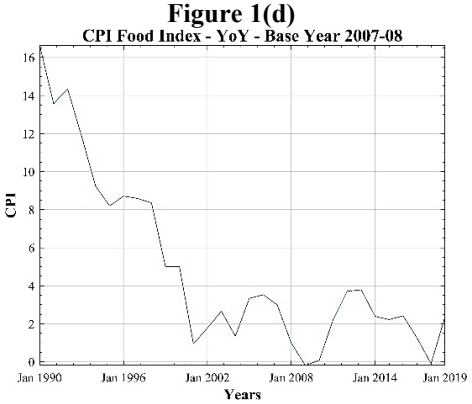


Figure 1(f)

Figure 1: Plots of Time Series of Variants of CPI (Figures 1(a)-1(f)

The plots are generated using a function of MatLab software where the values of CPI indices are represented on Y-axis and months/years are shown on X-axis. The movements of consumer prices indices over the selected time period can be visually observed from the respective plots.

After observation and analysis of the plots of time series of variants of CPI, the study moves to generate continuous wavelet transform of the selected variables for selected time period (Figure 2).

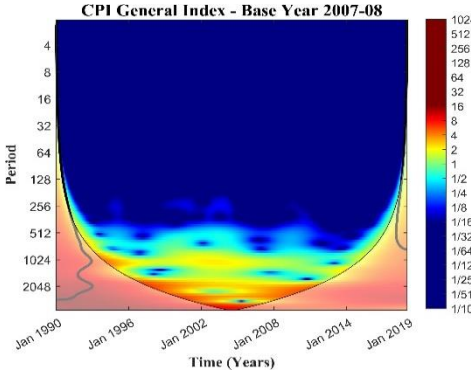
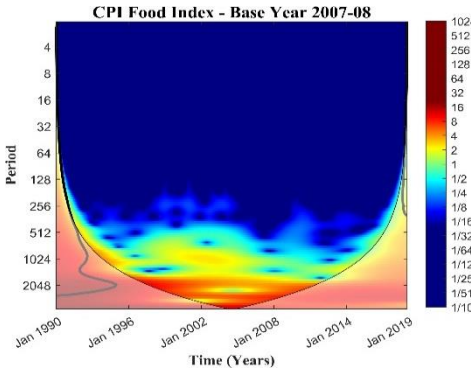


Figure 2(a)



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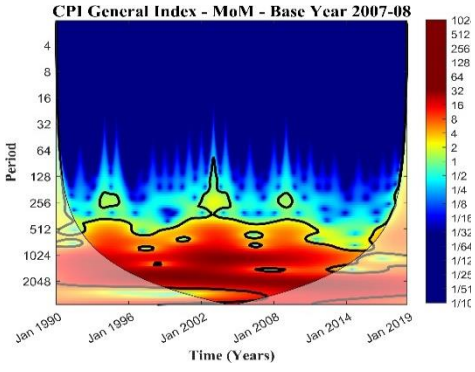

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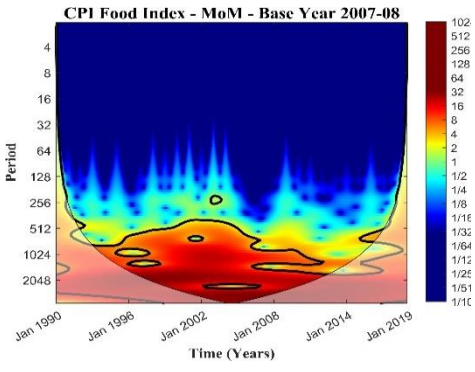

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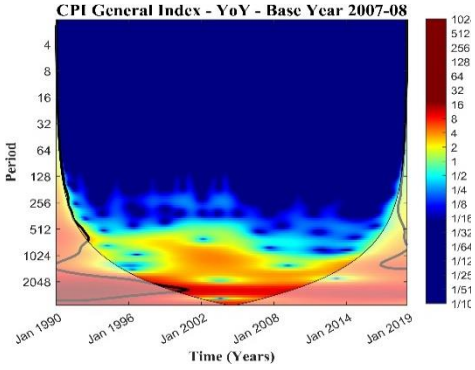

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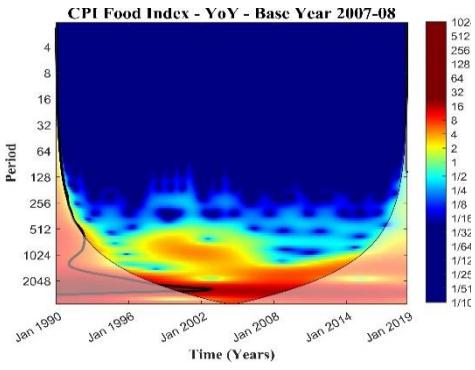


Figure 2(f)

Figure 2: Periodograms of Continuous Wavelet Transform

In the periodograms of CPI variants (Figure 2(a)-2(f), the horizontal axis represents time, whereas, the vertical axis represents scale (inversely related to frequency). Color scale represents wavelet coefficients magnitude blue to red (i.e. coarser coefficients to detailed coefficients). The bright regions in the periodogram indicate the presence of significant signal components at specific times and scales. Black thick curved line is Cone of Influence (COI), shaded area indicates 5% confidence level and shows edge-effects.

Having got the observation of and analysis of the continuous wavelet spectra of variants of CPI, the study moves to generate cross wavelet transform of the selected variables for selected time period (Figure 3).

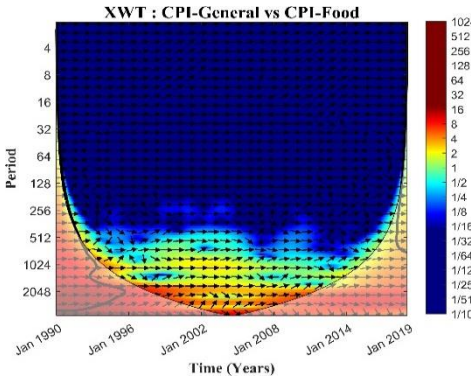
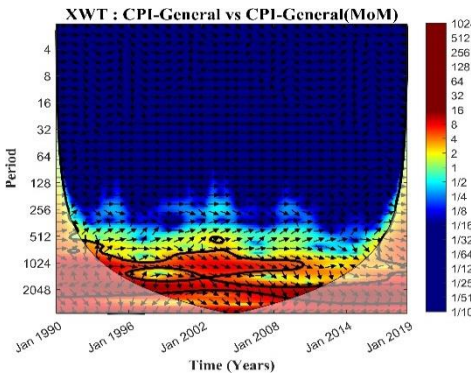


Figure 3(a)



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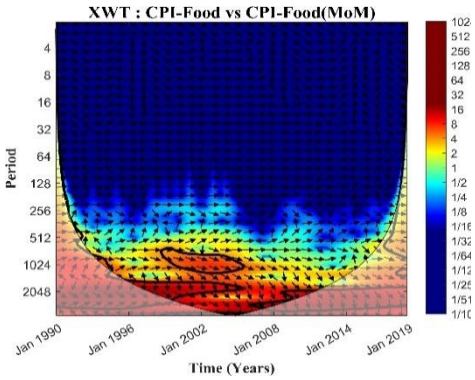

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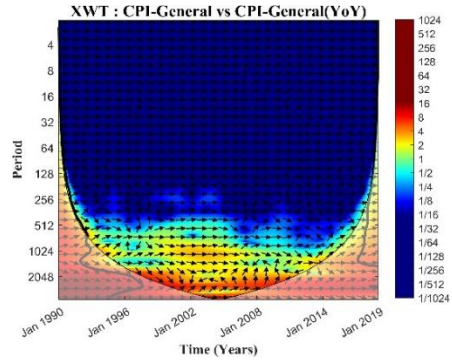

Figure 3(d)

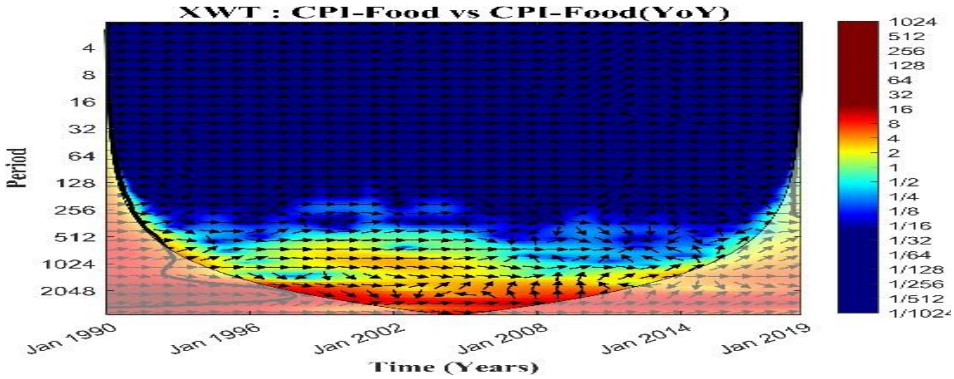

Figure 3(e)

Figure 3: Periodograms of Cross Wavelet Transform Figure 3(a)-3(e)

In the periodograms of CPI variants (Figure 3(a)-3(e), the horizontal axis represents time, whereas, the vertical axis represents scale (inversely related to frequency). Color scale represents wavelet coefficients magnitude blue to red (i.e. coarser coefficients to detailed coefficients). The bright regions in the periodograms indicate the presence of significant signal components at specific times and scales. Black thick curved line is Cone of Influence (COI), shaded area indicates 5% confidence level and shows edge-effects. The black thick arrows capture the co-variance of the contrasting variables and the phase information (i.e. exact-right arrows show that x and y are positively correlated, exact-left arrows show that x and y are negatively correlated, up-right arrows show that x and y are positively correlated and x is leading, down-right arrows show that x and y are positively correlated x is lagging, up-left arrows show that x and y are negatively correlated x is lagging, and down-left arrows show that x and y are negatively correlated x is leading).

Having got the observation of and analysis of the cross-wavelet spectra of variants of CPI, the study moves to generate wavelet coherence transform of the selected variables for selected time period (Figure 4).

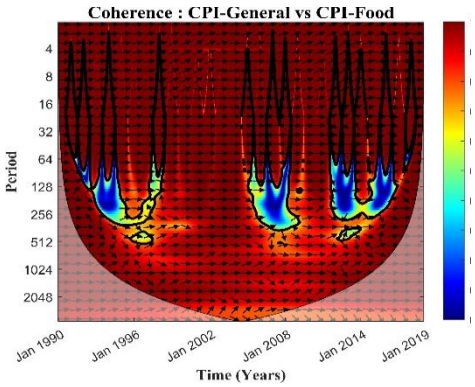

Figure 4(a)

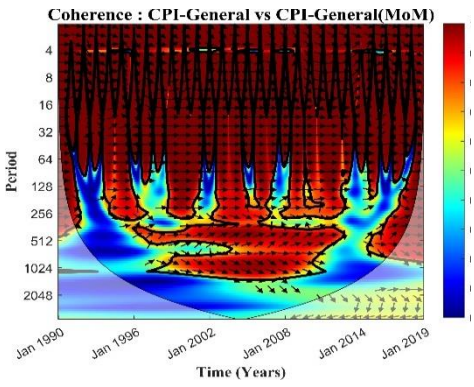

Figure 4(b)

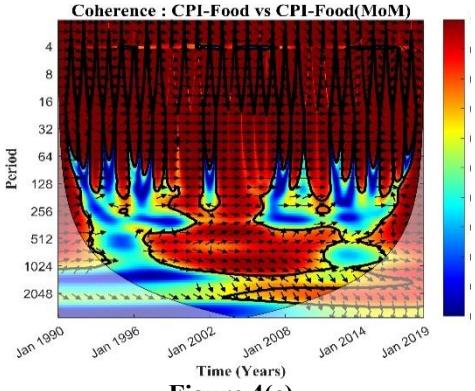

Figure 4(c)

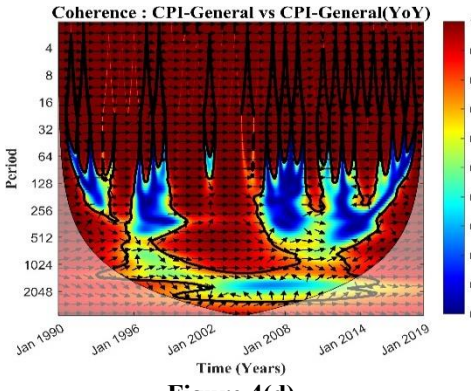

Figure 4(d)

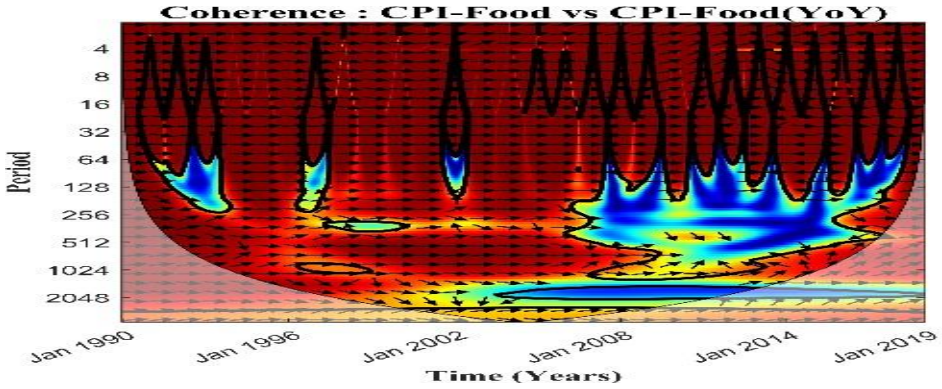

Figure 4(e)

Figure 4: Periodograms of Wavelet Coherence Transform Figure 4(a)-4(e)

In the periodograms of CPI variants (Figure 4(a)-4(e), the horizontal axis represents time, whereas, the vertical axis represents scale (inversely related to frequency). Color scale represents wavelet coefficients magnitude blue to red (i.e. coarser coefficients to detailed coefficients). The bright regions in the periodograms indicate the presence of significant signal components at specific times and scales. Black thick curved line is Cone of Influence (COI), shaded area indicates 5% confidence level and shows edge-effects. The black thick arrows capture the correlation of the contrasting variables and the phase information (i.e. exact-right arrows show that x and y are positively correlated, exact-left arrows show that x and y are negatively correlated, up-right arrows show that x and y are positively correlated and x is leading, down-right arrows show that x and y are positively correlated x is lagging, up-left arrows show that x and y are negatively correlated x is lagging, and down-left arrows show that x and y are negatively correlated x is leading).

4.3 Discussion

Admitting that CPI is one of important measure of the inflation of the countries, the study aims to examine times varying spectral coherence of CPI of Pakistan. Overall design being the data attainment, review of contemporary literature, and analysis of CPI General Index (Spliced Base 2007-08=100), CPI Food Index (Spliced Base 2007-08=100), CPI General MoM, CPI Food MoM, CPI General YoY, and CPI Food YoY from February 28, 1990 to 30th November, 2019 have been analyzed. Wavelet analysis along with classical descriptive statistics is used to analyze the variables. The analysis is represented above in analysis section.

Results of plots of time series of variants of CPI (Figure 1) reveal more frequent changes in prices as compared to the base year/month in the period ranging from January 1990 to January 2008 and after that there is relatively minor fluctuation in prices as compared to the base. In a wavelet-coefficient image obtained by wavelet analysis, horizontal axis represents distance, vertical axis represents spatial frequency, whereas coefficient magnitude on color scale indicates how strongly the data are correlated with the mother wavelet at a given frequency and distance.

The results of the wavelet power spectrum Figure 2(a)-(b) and (e)-(f) show that at frequency band 4-256 there is weak correlation between wavelets and mother wavelet, whereas, that is high at 256-2048 frequency bands throughout the selected period. However, the power spectra as contained in Figure 2(c)-(d) one can find the contours enveloped in thick black line horizontally appearing against late 1990s to 2014 at frequency bands 512-2048 depicting very strong correlation between the mother and daughter wavelets.

The results of the cross-wavelet spectra Figure 3(a), (d) and (e) (i.e. CPI-General vs CPI-Food, CPI-General vs CPI General (YoY), and CPI-Food vs CPI-Food (YoY)) show that at frequency band 4-256 there are weak signs co-movement of the variables, whereas, that

is high at 256-2048 frequency bands throughout the selected period. However, the cross-wavelet spectra as contained in Figure 3(b)-(c) (i.e. CPI-General vs. CPI-General (MoM), and CPI-Food vs. CPI-Food (MoM) at frequency bands 512-2048 have very strong co-movement on selected time scale.

The results of the wavelet coherence spectra Figure 4(a), (d) and (e) (i.e. CPI-General vs CPI-Food, CPI-General vs CPI General (YoY), and CPI-Food vs CPI-Food (YoY)) show that at frequency bands 4-256 there are signs of high coherence and correlation among the variables, whereas, that is relatively low at 256-2048 frequency bands throughout the selected period. However, the wavelet coherence spectra as contained in Figure 4(b)-(c) (i.e. CPI-General vs. CPI-General (MoM), and CPI-Food vs. CPI-Food (MoM) at frequency bands 512-2048 have weak coherence and correlation on selected time scale.

The results also show that both the variables are in phase at most of the frequency and time resolutions. It is worthwhile to discuss the results, implications, contributions, limitations and future recommendations by the study qua reality keeping mind the main objective of the study (i.e. to examine times varying spectral coherence of CPI of Pakistan). There is a plethora of researches on the phenomenon that the authors encountered while during searching the research data basis aforementioned in literature section. However, as result it is concluded that there is hardly any research that explored the phenomenon's time and frequency domain simultaneously. The study extends the scarce literature on CPI General Index (Spliced Base 2007-08=100), CPI Food Index (Spliced Base 2007-08=100), CPI General MoM, CPI Food MoM, CPI General YoY, and CPI Food YoY of Pakistan.

The study provides a framework to measure the frequency components of dynamic movement of these variables without losing time-specific information. It exposes spatial regions in terms of the degree and direction (in phase or out phase) of co-movement of different variants of CPI index. It also detects as to how the interrelationships between different variants of CPI index are evolving through time and across different frequencies.

The study has practical implications by way of providing better insights for marketers / businesses to project expenses and budgets, investors to make informed investment decisions, consumers to understand how their purchasing power is changing and to regulators to better formulate the economic policies. It also has theoretical implications by way of providing the theoretical framework for further studies.

4.4 Limitations and Future Directions

There are certain limitations of the study as well, therefore, findings of the study are accordingly useful. The wavelet analysis is shift sensitive, has relatively poor directionality, gives limited phase information and it is computationally intensive analysis and needs even more precision. Therefore, future researches should rather precise analysis based on an extended dataset by using multiple and partial wavelet coherency methodologies.

The future studies should construct other nonlinear models that measure asymmetry and can use the other advanced econometric technique to have a more comprehensive result. However, the study contributed towards a lot of new information about the phenomenon understudy viz descriptive statistics, plots, periodograms of continuous wavelet transform, cross wavelet transform and coherence wavelet transform, and the discussion on the results.

5. Conclusion

CPI is one of the most important economic hall marks that helps to gauge the health of the economy of the country. It is an ever-green area of research. In the context of Pakistan, CPI indices are studied by plenty of researchers but the authors could not find any study that uses this type of the novel approach and considered time and frequency domains both to measure co-movement of different variants of CPI general comparatively. Therefore, the current study is designed to study the CPI indices of Pakistan using the novel approach of wavelet analysis. The study combines the frequency-domain approach with time-domain approach through wavelet analysis (i.e. plots, wavelet individual power spectra, cross wavelet transform spectra, wavelet coherence spectra). Overall design is getting data, review of contemporary literature, and analysis. The data are obtained from the central bank of Pakistan. The variables include CPI General Index (Spliced Base 2007-08=100), CPI Food Index (Spliced Base 2007-08=100), CPI General MoM, CPI Food MoM, CPI General YoY, and CPI Food YoY.

The time series is February 28, 1990 to 30th November, 2019. Plots of time series of variants of CPI show that there are more frequent changes in prices as compared to the base year/month in the period ranging from January 1990 to January 2008 and after that there are minor fluctuation. Results of the wavelet power spectra of General Index (Spliced Base 2007-08=100), CPI Food MoM, CPI General YoY, and CPI Food YoY show that at frequency band 4-256 there is weak correlation between wavelets and mother wavelet, whereas, that is high at 256-2048 frequency bands throughout the selected period. In that of CPI Food Index (Spliced Base 2007-08=100), CPI General MoM, there is very strong correlation between the mother and daughter wavelets.

Results of the cross-wavelet spectra show that CPI-General vs CPI-Food, CPI-General vs CPI General (YoY), and CPI-Food vs CPI-Food (YoY)) at low frequency bands have weak co-movements, whereas, that is strong at high frequency bands throughout the selected period. Cross wavelet spectra of CPI-General vs. CPI-General (MoM) and CPI-Food vs. CPI-Food (MoM), at high bands have very strong co-movement. Results of the wavelet coherence spectra show that at low frequency bands there is high coherence and correlation among variables, whereas, that is relatively low at high frequency bands throughout the selected period. However, the wavelet coherence spectra as contained of CPI-General vs. CPI-General (MoM), and CPI-Food vs. CPI-Food (MoM) high frequency bands very weak coherence and correlation on selected time scale.

It is an original value study based on real-time time-series data and state of the art techniques of analysis. The finding of the study is useful for government of Pakistan to since it will be helpful for formulating monetary and fiscal policies. It will also be useful for marketers/businesses to project expenses and budgets, investors to make investment decisions, consumers to understand how their purchasing power is changing.

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